OPTIMUM HUB HEIGHT FOR WIND TURBINE INSTALLATIONS

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ABSTRACT. The annual energy production of a wind turbine is one of the most important parameters which determine the economical viability of such an installation. In the present work the influence of the hub height along with the topography effects are taken into account in order to maximize the efficiency of a wind turbine installation, without significantly increasing the installation cost. For this purpose long period measurements are used for three representative regions of the Aegean Archipelago, including an area with medium, an area with good and another one with excellent wind potential. As in many cases of interest for wind energy applications, it is necessary to estimate wind speeds and frequencies at hub height from measurements done at a different level, usually at 10m level. In order to increase the reliability of our results the influence of the "wind shear exponent" of the power law model along with the thickness of the atmospheric shear layer are taken into account. Additionally four different types of medium-large size wind turbines are used to check the effects of the machine's power curve type on the results obtained. Finally the calculated energy production results are summarized for all the combinations of areas, wind turbine types, hub heights and wind shear exponent values. These results are also compared with the first installation cost increase and interesting conclusions are drawn. By using the experience obtained from the present analysis a goal of more than 50% energy production rise may be achieved, with much less growth of the investment cost.

INTRODUCTION

The energy production of a wind turbine installation is one of the most important parameters who determine the economical viability of such an investment, [1]. Additionally the net energy output produced during a year depends strongly on the exact value of the mean power coefficient (or capacity factor). It is worthwhile to mention that according to previous research [2] a 0.1 increase of the value of the mean power coefficient results to a 60% decrease of the pay-back period of the investment. Therefore the amelioration of the economic attractiveness of similar applications by increasing the value of the mean power coefficient is one of the most challenging research problems in the wind turbine applications area.

In previous work [3] the importance of the appropriate interaction between the local wind potential and the operational characteristics of the wind turbine to be used is proved. In the present work the influence of the hub height along with the topography effects are taken into account in order to maximize the efficiency of a wind turbine installation, without significantly increasing the installation cost. For this purpose long period measurements are used for various representative regions of the Aegean Archipelago, including an area with medium, an area with good and another one with excellent wind potential. The experimental data consist of wind speed and frequency values for at least five years. However, as in many cases of interest for wind energy applications, it is necessary to estimate wind speeds and frequencies at hub height from measurements done at a different level, usually at 10m level. In similar situations several semi-empirical velocity profiles exist in the literature and in order to increase the reliability of our results the influence of the "wind shear exponent" of the power law model along with the thickness of the atmospheric shear layer are taken into account. Additionally four different types of medium size commercial wind turbines are used to check the effects of the machine's power curve type on the results obtained.

Finally the calculated energy production results are summarized for all the combinations of areas, wind turbine types, hub heights and wind shear exponent values. These results are also compared with the first installation cost increase and interesting conclusions are drawn. By using the experience obtained from the present analysis a goal of more than 50% energy production rise may be achieved, with much less growth of the investment cost.

ENERGY PRODUCTION

The annual energy production "E" by a wind turbine of nominal power "N_o" is:

$$E = 8760 \bullet \omega \bullet \Delta \bullet N_o \tag{1}$$

where "Δ" is the mean annual availability of the installation and "ω" is the mean power coefficient (capacity factor) defined as:

$$\omega = \int_0^\infty \frac{N(V)}{N_o} \bullet f(V) \bullet dV \tag{2}$$

For the calculation of the exact value of "ω" the wind turbine power curve "N=N(V)" provided by the manufacturer is utilized along with the experimental probability density function "f=f(V)" of the wind speeds. More precisely "ω" can be estimated using a more practical formulation, i.e:

$$\omega = \omega_1 + \omega_2 = \int_{V_c}^{V_R} \frac{N(V)}{N_o} \bullet f(V) \bullet dV + \int_{V_R}^{V_F} \frac{N(V)}{N_o} \bullet f(V) \bullet dV \tag{3}$$

and therefore is a function of the wind speed and direction as well as of the probability density function "f(V)" of the wind speed at the specific installation site and at the height of wind turbine's axis. Additionally, the mean power coefficient depends on the specific powercurve "N=N(V)" of the wind turbine and its exact value is the result of the interaction between the local wind potential and the operational characteristics of the machine, [3].

WIND TURBINE POWER CURVE

Table I: Main Wind Turbines' Characteristics

WIND TURBINE's CHARACTERISTICS	A	B	C	D
Nominal Power (kW)	270	250	250	250
Rotor diameter (m)	25.5	25	25.3	24
Hub height (m)	31.5	29.85	30	29.5
Cut in Wind Speed (m/s)	3	4	5	4
Rated Wind Speed (m/s)	13	14	14	14
Cut out Wind Speed (m/s)	25	24	21	22

A typical powercurve is composed by the transitional part, where the power increases with the increase of the wind speed from the cut in wind speed "V_c" to the rated one "V_R", and the (almost) constant power part as the wind speed takes values between the rated speed and the cut out speed "V_F". For theoretical analysis purposes one may write:

$$\begin{array}{ll} N = 0 & if(V \le V_c) \\ N = N(V) = a + b\bullet V \text{ or } a + b\bullet V + c\bullet V^2 & if(V_c \le V \le V_R) \\ N \approx N_o & if(V_R \le V \le V_F) \\ N = 0 & if(V \ge V_F) \end{array} \qquad (4)$$

therefore "ω_1" and "ω_2" can be expressed as:

$$\omega_1 = \frac{a}{N_o}\bullet(F(V \le V_R) - F(V \le V_c)) + \frac{b}{N_o}\bullet\int_{V_c}^{V_R} V\bullet f(V)\bullet dV \qquad (5)$$

where "$F(V \le V_o)$" is the probability function expressing the possibility to exist wind speed values less than a given value, e.g. "V_o". Similarly,

$$\omega_2 = F(V \le V_F) - F(V \le V_R) \qquad (6)$$

However, for more realistic calculation results one may use the exact powercurve of a machine as given by the manufacturer for standard atmospheric conditions. In the present work four representative powercurves of commercial machines are selected from the "WIND-BASE" [4] to be analyzed, after their operational characteristics are modified to include local atmospheric conditions. In fig.1 the powercurves of these four machines are given, while some useful characteristics of them are summarized in Table I. The wind turbines investigated are either stall or pitch control.

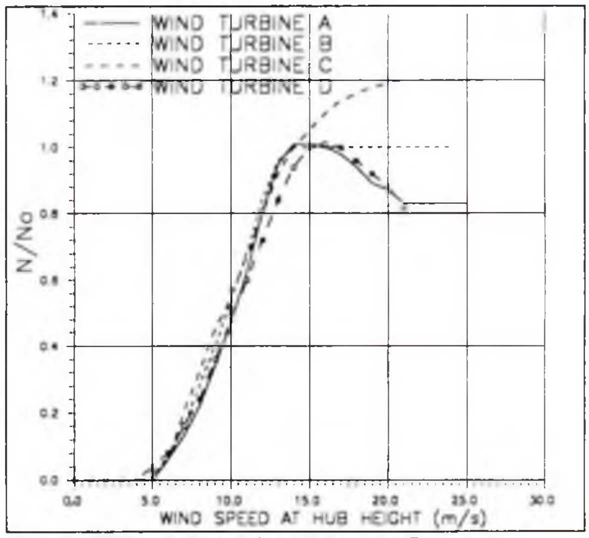

Fig.1: Wind Turbine Power Curves

WIND VELOCITY PROFILES

For the description of the wind potential of an installation site the wind speed and frequency are necessary, referred at wind turbine's hub height. However, in most cases the wind data are given at standard height, usually at 10m from the ground. In these cases some semi-empirical velocity profiles are adopted from the literature and the most common of them are the "power law" and the "log law". According to the existing analysis [5] the log law provides the closest fit in the 30÷50m height range but throughout the atmospheric boundary layer height, the power law is more accurate. Additionally because its simplicity, the power law is more often used, i.e:

$$\begin{array}{ll} \dfrac{V}{V_1} = (\dfrac{h}{h_1})^a & for(h \le \delta) \\ V = V_\delta & for(h \ge \delta) \end{array} \qquad (7)$$

where the velocity value "V_1" is measured at "$h=h_1$" (usually at 10m) and the exponent "a" is principally determined by the thermal stratification of the air and the surface roughness

Various researchers suggest different values of "a" is many cases. For example Mikhail and Justus [6] propose the following formulae:

$$a = \frac{0.37 - 0.088\ln[V(h = h_1)]}{1 - 0.088\ln(h_1/10)} \qquad (8)$$

while for quick calculation they propose that a=0.23±0.03. Other researchers adopt smaller values, like Delaney [7] who uses the value a=0.1 for the Brittany region and Williams [8] who utilizes the value 0.081. Generally speaking the wind shear exponent takes values between 0.05 for very smooth areas (e.g. sea, sand, snow) up to 0.45 for very rough areas (e.g. urban areas, tall buildings). Besides, according to a recent research by CRES in Crete [9] it was found that the shear exponent exhibits a considerable variability with the site and the direction sector, taking values from slightly negative (accelerating flow, jet like profiles) to positive up to 0.16.

Finally there have been established some interesting relations between the surface roughness "h_o" and the shear exponent "a", see for example the one proposed by Warne and Calnan [10], i.e.

$$a = 0.04\ln(h_o) - 0.003\,[\ln(h_o)]^2 - 0.24 \tag{9}$$

WIND SPEED FREQUENCIES

The wind speed probability density function "f(V)" gives the possibility to exist wind speed values in a very narrow area of "V", namely "V±dV". Therefore the possibility of the wind speed to be between "V_a" and "V_b" is given as:

$$p(V_a \le V \le V_b) = \int_{V_a}^{V_b} f(V)dV = F(V \le V_b) - F(V \le V_a) \tag{10}$$

Using eq.(7) to describe the velocity profiles of the wind we may assume that the possibility to encounter wind speed values at a given height "h_1" in the area of (V_a,V_b) is the same with the possibility to appear wind speeds at another height "h" in the area $[V_a\,(h/h_1)^a, V_b\,(h/h_1)^a]$. In the case that the velocity interval (V_b-V_a) is very small we get from equation (10) that:

$$f_h = f_{h_1} \bullet \left(\frac{h_1}{h}\right)^a \tag{11}$$

Taking into account the above equation one may understand why is not valid that *the energy capable of being intercepted is proportional to* "h^{3a}" and that the continuous increase of the hub height of a wind turbine will lead to an analogous increase of the output power of the installation. In similar cases keep in mind that the wind rotor is still working in the same velocity region (e.g. from $V_c \approx 4$m/s to $V_F \approx 25$m/s).

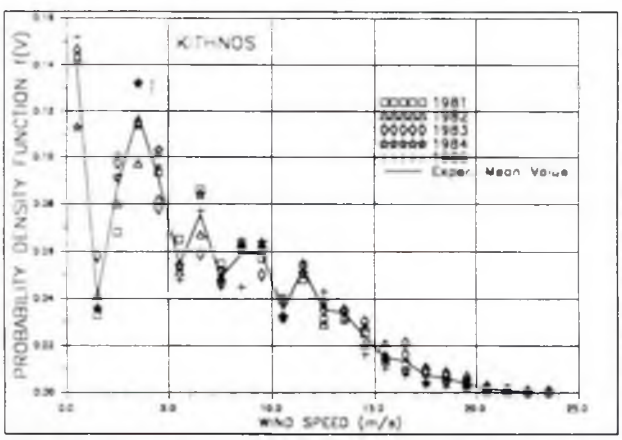

Fig.2: Experimental Wind Speed Data.

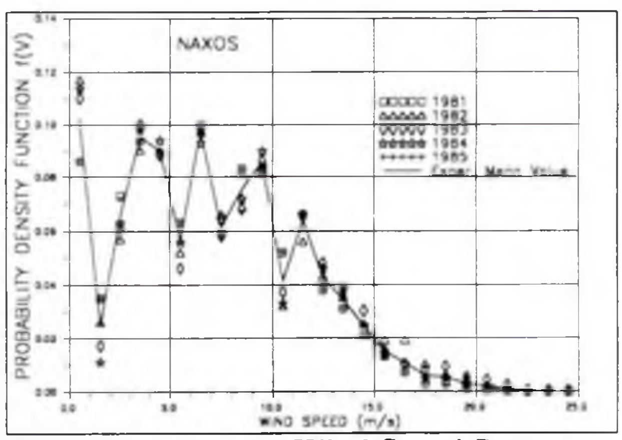

Fig.3: Experimental Wind Speed Data.

In the present work long period measurements [12] (for five years) are used for three representative regions of the Aegean Archipelago in order to examine the influence of the hub height on the energy production for these typical cases with totally different wind speed pattern. The areas under investigation are the island of Kithnos which has a medium quality wind potential (C=6.52, k=1.33), the island of Naxos as an example of an area with good wind potential (C=7.26, k=1.50) and the island of Andros which has excellent wind potential (C=9.83,k=1.86). Note that "C" and "k" are the well known Weibull's parameters [11], related with the local wind speed (potential) characteristics. The probability density function of these areas, based on measurements [12] at 10m height for a five year period, are given in figures 2 to 4, along with the five years experimental mean value. These experimental data are to be combined in the following with the power curves of the four wind turbines of fig.1, given by the manufacturers and properly modified to include local atmospheric conditions, in order to estimate the exact values of the mean power coefficient. Finally, in fig.5 the influence of the hub height upon the experimental value of the probability density function, for the island of Andros, is presented assuming medium rough topography, i.e. a=0.2. As it is obvious from fig.5 the "f(V)" values become smaller with increasing hub height, while the corresponding "f(V)" distributions are moving towards larger wind speeds.

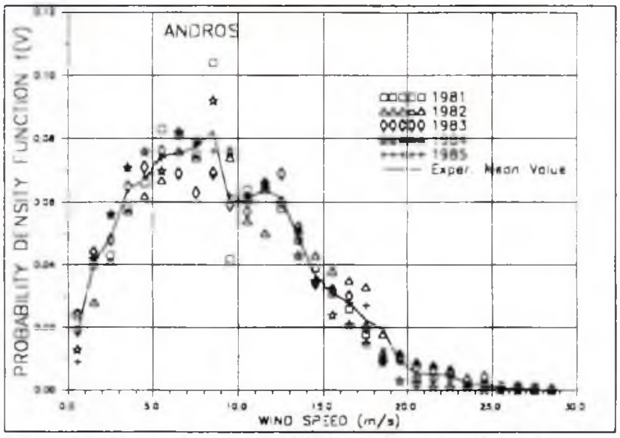

Fig.4: Experimental Wind Speed Data.

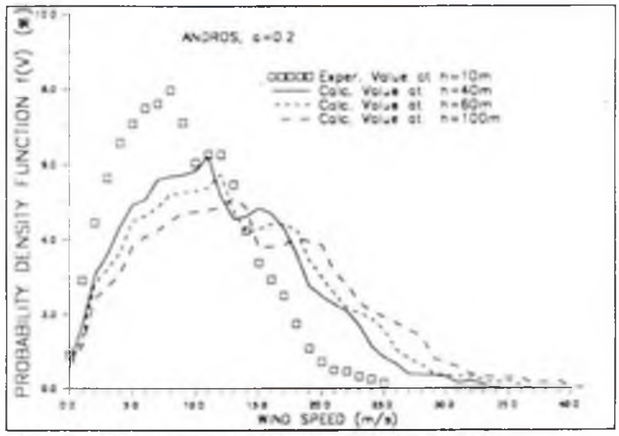

Fig.5: Change of "f(V)" with Height.

CALCULATION OF ENERGY PRODUCTION AT DIFFERENT HUB HEIGHTS

For the calculation of the mean power coefficient and the corresponding energy production equations (1) to (3) are used for every combination of installation site and wind turbine. For each combination (3x4) the influence of the hub height is to be examined, using as a parameter the value of the wind shear exponent "a" and the atmospheric boundary layer height. At this point the atmospheric boundary layer thickness is taken equal to δ=200m.

Island of Naxos

Using the experimental mean probability density function of Naxos (fig.3) for the period 1981-1985 properly modified according to equation (11) for each one hub height value investigated we calculate the evolution of the mean power coefficient "ω" versus height, i.e. ω=ω(h), for typical values of "a" and for all four wind turbines used. According to the results given in figures 6a to 6d one may conclude that the mean power coefficient is strongly depended on the hub height. More precisely, for all the machines examined and for small values of "a" (a≤0.1) there is a remarkable increase of "ω" with the height, especially for the first 40m. The increase of "ω" is much more greater for a=0.2, but after the h=60m "ω" becomes almost constant. Finally, for the extreme value of "a" the maximum value of "ω" is predicted at h≈40m, while for bigger values of "h" ,"ω" decreases dramatically.

Island of Kithnos

The same behaviour is encountered also for the mean power distribution versus hub height for the island of Kithnos (fig.6e), although the numerical values predicted are quite smaller than the corresponding ones for Naxos. This fact was expected since Naxos has better wind potential than Kithnos. For both islands "ω" presents the biggest increase when the wind shear exponent takes the value a=0.2, a value often detected in similar conditions with short grass and rural areas.

Fig.6: Evolution of the Mean Power Coefficient "ω" with hub height.

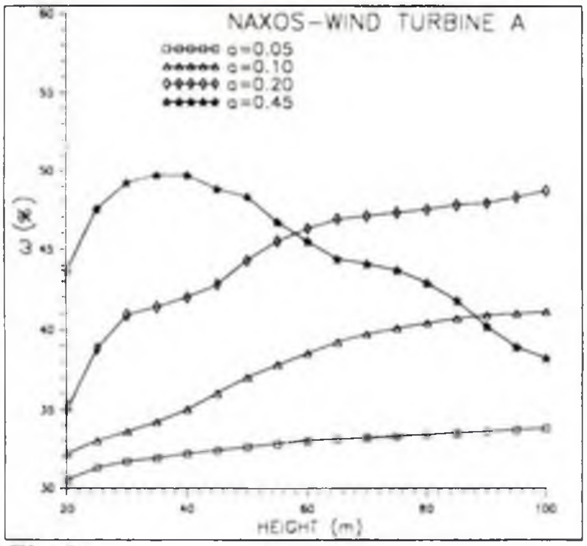

Fig.6a:

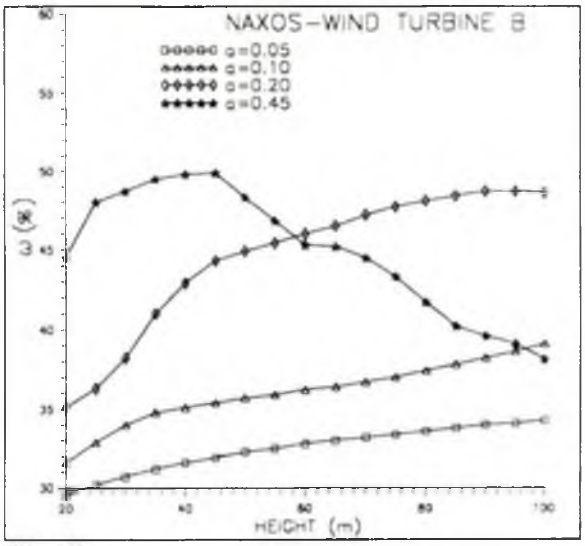

Fig.6b:

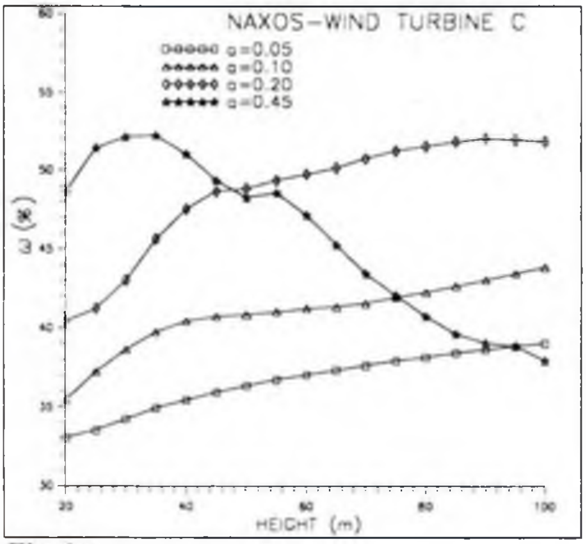

Fig.6c:

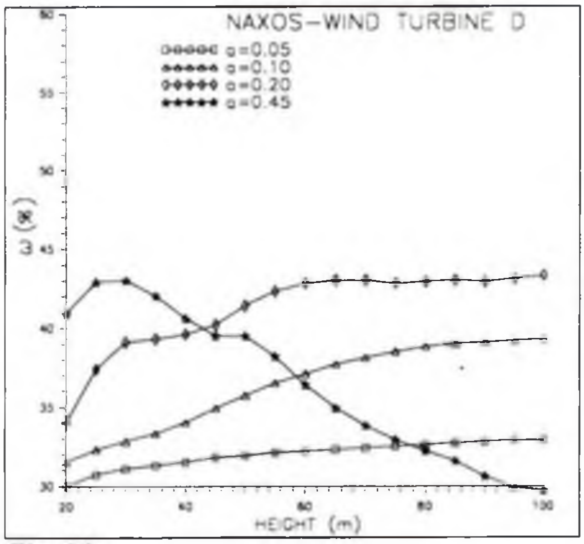

Fig.6d:

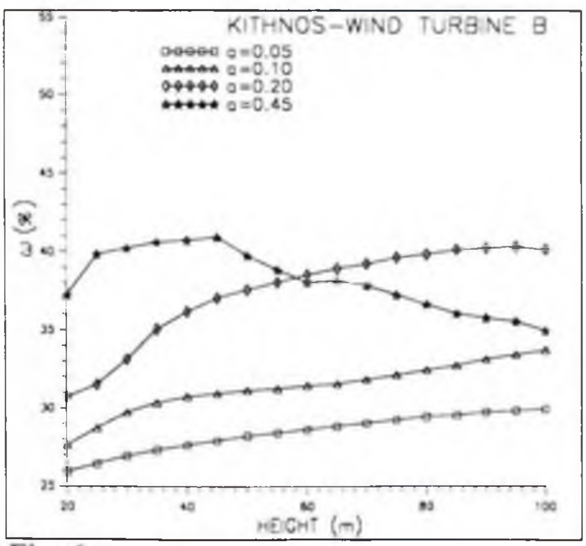

Fig. 6e:

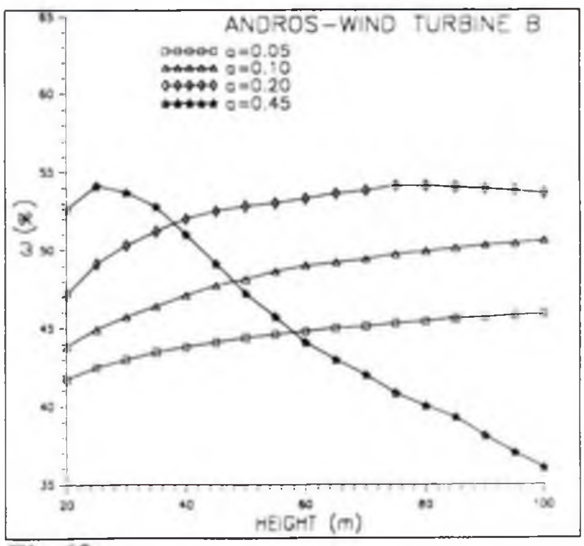

Fig.6f:

Island of Andros

Finally, the picture of Andros is slightly different, mainly due to the excellent wind potential of the island. According to our analysis (see fig.6f) the increase of "ω" with the height is much smaller in comparison with the other cases, and very soon "ω" becomes almost constant. Additionally, for the extreme value of "a" the mean power coefficient decreases monotonically after the first 25m. This fact is in accordance with our knowledge that in Andros the wind speeds are very high and they are continuous blowing, therefore any additional increase of wind speed with height can not be exploited by the existing commercial wind turbines.

Summarizing we can say that according to our data there is a remarkable increase of "ω" and of the corresponding energy production by a wind turbine installation with the hub height, especially for areas with medium wind potential and medium rough topography. In these cases the annual energy production may be increased by almost 25% with a parallel elevation of hub height from 25m to 50m. On the contrary in cases with very rough topography the proposed height is between 25m and 40m, since a remarkable decrease in energy production is detected with bigger heights. Finally, for areas with very good wind potential there is no need for the hub height to overpass the 40m, especially for sites with rough or very rough topography.

INFLUENCE OF MEAN POWER COEFFICIENT ON WTs' ECONOMIC VIABILITY

For the calculation of the pay-back period "n*" of the investment under consideration the complete break-even equation [13], has to be solved, i.e:

$$C_n = R_n \tag{12}$$

For the exact solution of equation (12) it is necessary to determine the accurate values of all parameters appearing in the break-even equation.

Investment Cost

The future value (after -n years) of the investment cost of a WT installation is a combination of the initial installation cost and the corresponding maintenance and operation cost. The initial investment cost includes the market and installation price of a wind turbine along with the cost of the necessary electrical and electronic equipments, i.e: "**$P_r N_o (1+f)$**". The maintenance and operation (M&O) cost can be split into the fixed maintenance cost "F_n" and the variable one "V_n". Expressing the annual fixed maintenance cost as a fraction (m%) of the initial capital invested and assuming that the annual increase in this cost is equal to the inflation rate "g", the future value of the fixed maintenance cost is given as:

$$F_n = m \bullet P_r \bullet N_o \bullet (1+i)^{n-1} \bullet (1+g) \bullet [1+(\frac{1+g}{1+i})+\ldots+(\frac{1+g}{1+i})^{n-1}] \tag{13}$$

The variable maintenance and operation cost depends mainly on the replacement of "k_o" major parts of the installation which have shorter lifetime "k_{max}" than the complete installation. Using the symbol "r_i" for the replacement cost of each one of the "k_o" major parts, the variable maintenance and operation cost can be expressed as:

$$V_n = P_r \bullet N_o \bullet \sum_{k=1}^{k=ko} r_k \bullet \sum_{l=1}^{l=l_k} (1+g)^{l \bullet k_{max}} (1+i)^{(n-l \bullet k_{max})} \quad (k = 1 \div k_o) \tag{14}$$

where "l_k" is the integer part of the following equation, i.e:

$$l_k = [(n-1)/k_{max}] \tag{15}$$

Using equations (13) to (15) the total investment cost "C_n" of the installation reads:

$$C_n = P_r \bullet N_o \bullet (1+f) \bullet (1+i)^n + V_n + F_n \tag{16}$$

Investment Revenue

The net energy output produced during a year by a wind turbine with nominal power "N_o"

is given by equation (1). Subsequently, the total energy produced by the wind turbine are given as:

$$R_n = \sum_{j=1}^{n} E_j \bullet c_j \bullet (1+i)^{n-j} \tag{17}$$

where "c_j" is the effective cost coefficient of the replaced conventional energy by the energy produced by the WT at the j-th year. According to Kaldellis (1991) the effective cost coefficient is directly related [14] with the fuel escalation rate (i.e. the annual rate of change of the fuel prices) via the following equation:

$$c_j = c_o \bullet \prod_{k=1}^{j} (1+e_k) \tag{18}$$

Using a mean fuel escalation rate value "e" equation (18) reads:

$$c_j = c_o \bullet (1+e)^j \tag{19}$$

savings over an -n year period due to the

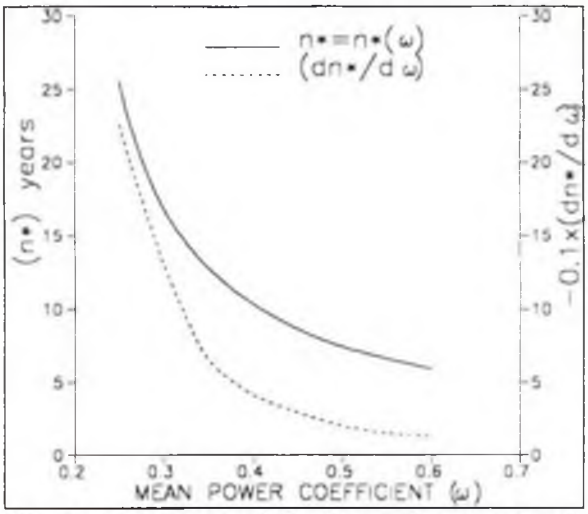

Fig.7: Influence of "ω" on the pay-back period of a WT installation.

Substituting equation (19) into equation (18) the total savings of the WT installation can be written as :

$$R_n = 8760 \bullet \Delta \bullet \omega \bullet c_o \bullet N_o \bullet \sum_{j=1}^{n} [(1+i)^{n-j} \bullet (1+e)^j] \tag{20}$$

Finally, using typical values for all the parameters of equations (12),(16) and (20) it is possible to estimate the pay-back period of a WT investment. In fig.7 the influence of the mean power coefficient on the pay-back period of a WT installation is given along with the first derivative "dn*/dω". As obvious, the pay-back period is strongly affected by the increasing values of "ω", especially when it is increased from 0.3 to 0.4. For bigger values of "ω" (better wind potential) the diminution of the pay-back period is moderate.

OPTIMUM HUB HEIGHT PREDICTION

Using equation (1) one may relate the change of "ω" to the change of annual energy production and subsequently to the annual investment revenue via equation (20), i.e:

$$\frac{(dR/dh)}{R} = \frac{(dE/dh)}{E} = \frac{(d\omega/dh)}{\omega} \tag{21}$$

According to equation (21) every relative change of the mean power coefficient with the height is directly analogous to an equal change of the annual investment revenue. Similarly, using equation (16), every relative increase of the initial investment cost "$\mathbf{C_o=Pr x N_o}$" is directly translated to an equal increase of annual investment cost, i.e:

$$\frac{(dC/dh)}{C} = \frac{(dC_o/dh)}{C_o} \tag{22}$$

Using the results of a recent market survey [4], the increase of the initial installation cost of a wind turbine with the increase of the tower (hub) height is estimated between 1% and 3% for every five meters. More accurately, the bigger raise appears at the first 40m, while for bigger towers the increase tends to 1%.

In order to get a good estimate about the best hub height of a wind turbine installation it is easy to conclude by directly comparing the change of the annual investment cost and revenue. Therefore in any case that the change of the revenue with the hub height is bigger than the corresponding change of the investment cost it is suggested to use higher towers, if available.

Applying the above presented analysis for the island of Kithnos (a=0.2, medium rough topography) for all four wind turbines tested we can see from fig.8 that for the machines "A" and "D" the optimum hub height "H" must be less than 31m approximately. The corresponding values by the manufacturers are 31.5m and 29.5 respectively. Another possible solution is for H=50m. On the other hand for the pitch control machines "B" and "C" the best solution is by far near H=35m, while H must be less than 41m at any case. For these two wind turbines the hub heights suggested by the manufacturers are 29.85m and 30m respectively. For this last case by increasing the hub height by 5m only we expect a by more than 6% elevation in our annual revenues, accompanied by a much smaller increase (1% to 3%) in the initial investment cost. Finally, using the analysis of equation (12) the corresponding decrease of the pay-back period is almost 1.5 years. However, for the same island but using the assumption of a very smooth topography (a=0.1, see fig.9) there is no need to increase the "H" more than 31m for the machines "B" and "C", while for the machines "A" and "D" the hub height should have its minimum value (diameter "D" of the rotor), i.e.

$$H \rightarrow H_{min} \approx D \cdot 1.0m \quad (23)$$

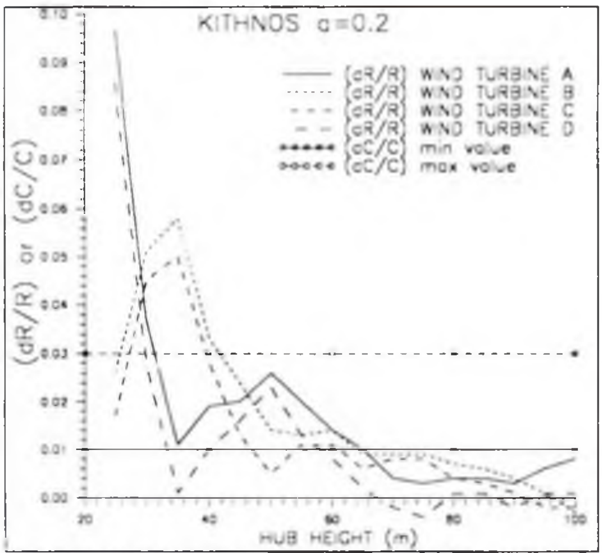

Fig.8: WT's relative cost and revenue changes with hub height.

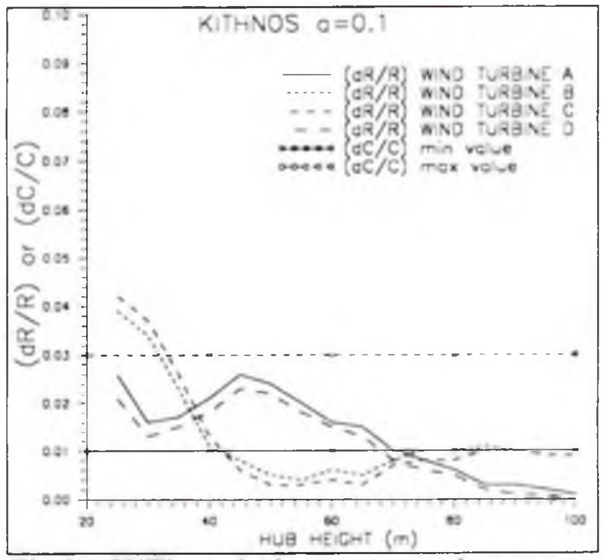

Fig.9: WT's relative cost and revenue changes with hub height.

Similar results are predicted for the island of Naxos also. In cases of medium rough topography the hub height of the stall control machines should be less than 34m, except in the case that H=50m. On the other hand the optimum hub height for machines "B" and "C" is approximately 37m. For the very smooth topography case the maximum "H" value suggested is 32m for "B" and 36m for wind turbine "C" respectively. Finally, for the island of Andros, due to its very good wind potential, the optimum hub height is near the technical minimum (see eq.(23)) for the topographies analyzed, e.g. fig.10. In this case any increase of energy production with the hub height is balanced by the corresponding increase of the initial cost.

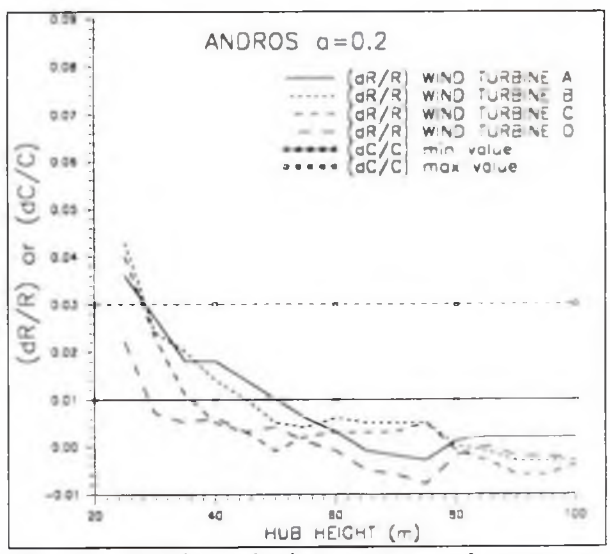

Fig.10: WT's relative cost and revenue changes with hub height.

CONCLUSIONS

A first systematic attempt to estimate the influence of the hub height of a wind turbine upon the annual energy production is presented. During this study a large variety of interesting examples is investigated, including cases with medium, good and excellent wind potential and other ones with smooth, medium rough or very rough topography of the installation site. According to the results obtained there is no need to increase the hub height of a wind turbine only for the areas with excellent wind potential or with very smooth topography. On the contrary, for all the other cases any increase of the hub height (up to $\approx$ 50m) will lead to a significant increase of the annual energy production. The conclusions mentioned above are valid for all the types of the machines analyzed, although some differences exist between the stall and the pitch control machines.

Recapitulating, it is worthwhile to mention that according to the proposed analysis, it is quite important to calculate the influence of the hub height increase on the energy production of any wind turbine installation, based on experimental wind data and real machine's power curves if possible. By using similar analysis a goal of more than 50% energy production rise may achieved, which should ameliorate the economic attractiveness of the wind turbine applications and increase the efficiency of the capital invested.

REFERENCES

[1] **Kaldellis J., Kodossakis D.,1991**. "The Economic Viability of Small Size Wind Turbine Installations, for the Greek Socio-Economic Environment. The Impact of the Technological Improvements and the Inflation Rate". 3rd European Symposium on "Soft Energy Sources and Systems at the Local Level", Chios.

[2] **Kodossakis D., Kaldellis J., 1993**. "The Impact of the Techno-Economic Factors on the Economic Viability of SSWT Installations. A Complete Sensitivity Analysis for the Greek Market", presented at the ASME Energy Systems and Ecology "**ENSEC'93**" Conference, Poland, 1993.

[3] **Kaldellis J., Blahou D., 1992**. "Economic Viability of Small Size Wind Turbine Installations. The Interaction Between the Local Wind Potential and the Operational Characteristics of the Wind Turbine". ASME Int. Conference on Efficiency, Costs, Optimization and Simulation of Energy Systems, Spain.

[4] **Kaldellis J., Arrianas L., Konstantinou P., Blahou D., 1996**. "Investigation of the Aerodynamic Behaviour of Commercial Wind Turbines", 5th National Conference on Soft Energy Resources, Greece.

[5] **Le Gourieres D., 1982**. "Wind Power Plants. Theory and Design", edition of Pergamon Press, Oxford.

[6] **Mikhail A.S., Justus C.G., 1981**. "Comparison of height. Extrapolation models and sensitivity analysis". Jr. of Wind Engineering, Vol.5, pp.91-106.

[7] **Delauney D., 1986**. "Wind energy assessment of the Brittany region", Proceedings EWEC'86, pp.333-337.

[8] **Williams G.J., 1988**. "A survey of the wind energy resource of Cornwall to examine the influence of settlement patterns and topography on optimum wind turbine size and disposition". PhD Thesis, Open University, Energy and Environment Research Unit, July 1988.

[9] **Glinou G., Foussekis D., Zorlos P., Panourgias J., 1995**. "Wind measurements in complex terrain. The case of Crete", Proceedings EWEC'95.

[10] **Warne D., Calnan P., 1977**. "Generation of Electricity from the wind", Proceedings IEE, Vol.124.

[11] **Kovarik T.,Pipher Ch.,Hurst J.,1979**. "Wind Energy", Domus Books, Chicago.

[12] **Greek Public Power Corporation, 1981-1985**. "Wind Potential Measurements Throughout Greece", edition of P.P.C., Athens.

[13] **United Nations Publication, 1978**. "Manual for the Preparation of Industrial Feasibility Studies", ID/206, New York.

[14] **Kaldellis J., 1991**. "Cost-Benefit Analysis Concerning the Small Scale Wind Turbines Installations for the Greek Socio-Economic Environment, Including Fuel Escalation", 2nd Conf. on Environmental Science & Technology, Lesvos, Greece.